Student Number: ______________________ Class Teacher: ______________________

St George Girls High School

Trial Higher School Certificate Examination

2018

Mathematics
Extension 1

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using black pen.
- Board-approved calculators may be used.
- A reference sheet is provided.
- In Questions 11 – 15, show relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for incomplete or poorly presented solutions.

Section I	/10
Section II	
Question 11	/12
Question 12	/12
Question 13	/12
Question 14	/12
Question 15	/12
Total	**/70**

Total Marks – 70

Section 1 Pages 3 – 6

10 marks

- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.
- Answer on the multiple choice answer sheet provided at the back of this paper.

Section II Pages 7 – 13

60 marks

- Attempt Questions 11 – 15.
- Allow about 1 hour and 45 minutes for this section.
- Begin each question in a new writing booklet.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the size of the acute angle between $y = 3x$ and $y = -2x + 1$, correct to the nearest degree?

(A) $36°$

(B) $135°$

(C) $11°$

(D) $45°$

2 Simplify $\dfrac{\cos 3\alpha}{\sin \alpha} + \dfrac{\sin 3\alpha}{\cos \alpha}$.

(A) $\tan 2\alpha$

(B) $\cot 2\alpha$

(C) $2\tan 2\alpha$

(D) $2\cot 2\alpha$

3 What is the maximum value of the function $y = \sin x - \sqrt{3}\cos x + 1$.

(A) 1

(B) 2

(C) 3

(D) 4

4 The area of a rectangular bar is $A = 50L - L^2$, where L is the length of the bar. The bar is heated and its length increases at the rate of 0.08 cm/min. At what rate is the area of this bar increasing when $L = 15$cm?

(A) 3.6 cm²/min

(B) 2.4 cm²/min

(C) 1.6 cm²/min

(D) 0.8 cm²/min

5 Find the value of a, if $(x + a)$ is a factor of $P(x) = x^3 + ax^2 + 2x + 1$.

(A) $-\frac{1}{2}$

(B) ± 1

(C) $\frac{1}{2}$

(D) $\pm\frac{1}{\sqrt{2}}$

6 If $y = \pi^x$, find $\frac{dy}{dx}$ when $x = 1$.

(A) 1

(B) π

(C) $\pi \ln \pi$

(D) $x\pi - \ln x$

7 What is the exact value of $\int_0^4 \frac{dx}{x^2+16}$?

(A) $-\frac{\pi}{4}$

(B) $\frac{\pi}{16}$

(C) $\frac{\pi}{8}$

(D) $\frac{\pi}{4}$

8 Which of the following is the range of the function $= 2\sin^{-1} x + \frac{\pi}{2}$?

(A) $y \in \mathbb{R}: -\pi \leq y \leq \pi$

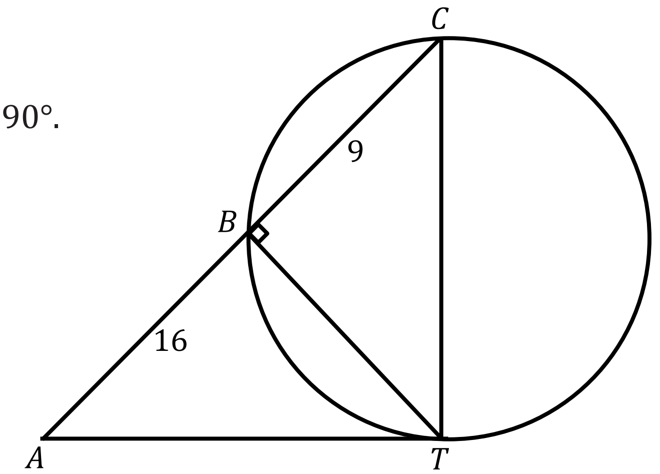
(B) $y \in \mathbb{R}: -\pi \leq y \leq \frac{3\pi}{2}$

(C) $y \in \mathbb{R}: -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(D) $y \in \mathbb{R}: -\frac{\pi}{2} \leq y \leq \frac{3\pi}{2}$

9 AT is tangent to the circle at T.
AC cuts the circle at B.
$AB = 16$, $BC = 9$, and $\angle TBC = 90°$.

Which is the length of TC, correct to the nearest whole number?



(A) 9

(B) 15

(C) 22

(D) 25

10 Which of the following is an expression for $\tan(\cos^{-1} x)$?

(A) $\sqrt{1-x^2}$

(B) $\dfrac{\sqrt{1-x^2}}{x}$

(C) $\dfrac{x}{\sqrt{1+x^2}}$

(D) $\dfrac{\sqrt{1+x^2}}{x}$

End of Section I

Section II

60 marks
Attempt Questions 11 – 15
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 **(12 marks)** **Use a separate writing booklet** **Marks**

(a) Solve $|x-7| < |3-x|$. 2

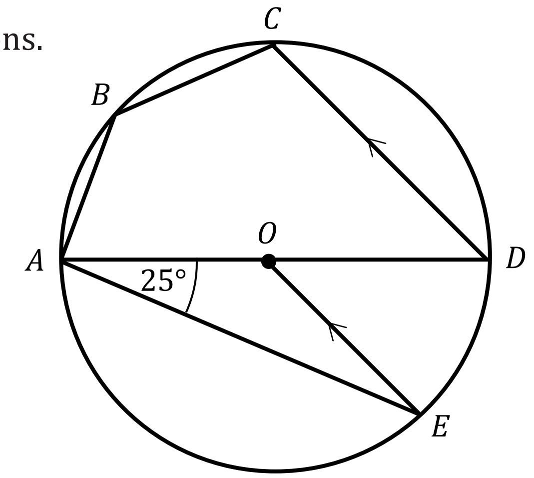
(b) Find $\dfrac{d}{dx}(x\sin^{-1}x)$. 2

(c) The point R divides the interval from $A(-10,8)$ to $B(-5,-1)$ externally in the ratio 1:2. 2
Find the coordinates of R.

(d) Find $\displaystyle\int \sec^2 x \tan x \; dx$. 1

(e) Evaluate $\displaystyle\lim_{x\to 0}\frac{\sin(\pi+x)}{x}$. 2

(f) The points A, B, C, and D lie on a circle with centre O, such that AD is a diameter. The point E lies on the circle so that OE is parallel to CD. $\angle OAE = 25°$. 3
Find the size of $\angle ABC$, giving reasons.



Question 12 (12 marks) Use a separate writing booklet **Marks**

(a) Solve $\dfrac{4x-1}{x+2} \geq 1$. 3

(b) Use the substitution $u = x + 1$ to find the value of 4

$$\int_0^3 \frac{x}{\sqrt{x+1}}\,dx.$$

(c) The velocity v m/s of a particle moving in simple harmonic motion along the x-axis is given by $v^2 = 6 + 4x - 2x^2$, where x is in metres.

i) Between which two points is the particle oscillating? 1

ii) Find the maximum speed of the particle in exact form. 1

iii) Find the acceleration of the particle in terms of x. 1

(d) Find the general solution of $2\sin^2 x - 1 = 0$.
Give your answer in radians. 2

Question 13 (12 marks) Use a separate writing booklet **Marks**

(a) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ (where $p > q > 0$) lie on the parabola $x^2 = 4ay$.

i) Derive the equation of the tangent to the parabola at P. **2**

ii) Find the coordinates of the point of intersection T of the tangents to the parabola at P and Q. **2**

iii) Show that if the tangents at P and Q intersect at 45°, then $p - q = 1 + pq$. **1**

iv) Find the locus of T by evaluating the expression $x^2 - 4ay$ at T and using the result in part (iii). **2**

(b) Express $2\sin x - 3\cos x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 \leq \alpha \leq 2\pi$. Give the value for α to 1 decimal place. **3**

(c) At time t years after the start of the year 2000, the number of individuals in a population is given by $N = 80 + Ae^{0.1t}$, for some constant $A > 0$. **2**

If there were 100 individuals in the population at the start of the year 2000, during which year is the population expected to reach 200?

Question 14 **(12 marks)** **Use a separate writing booklet** **Marks**

(a) i) Use the sum of the terms of an arithmetic series to show that 1

$$(1+2+3+\cdots+n)^2 = \frac{1}{4}n^2(n+1)^2.$$

ii) Prove the following expression by mathematical induction: 3
$1^3 + 2^3 + \cdots + n^3 = (1+2+\cdots+n)^2$ for all integers $n \geq 1$.

(b) i) Divide the polynomial $f(x) = 2x^4 - 10x^3 + 12x^2 + 2x - 3$ by $g(x) = x^2 - 3x + 1$. 2

ii) Hence express $f(x)$ in the form $f(x) = g(x)q(x) + r(x)$, where $q(x)$ and $r(x)$ are polynomials, and $r(x)$ has degree less than 2. 1

iii) Hence show that $f(x)$ and $g(x)$ have no zeroes in common. 1

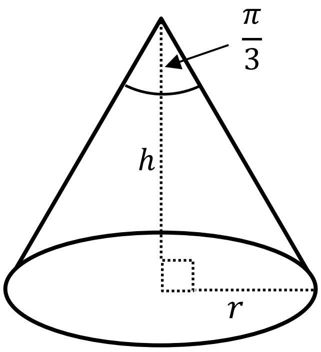
Question 14 continues on next page

Question 14 continued **Marks**

(c) Coal is poured at a constant rate of 1.5 cubic metres per second from a ship onto a conical pile on a dock.

The angle at the apex of the cone is a constant $\frac{\pi}{3}$ radians.

At time t seconds the height of the cone is h metres and the radius of the base is r metres.



(i) Show that $r = \dfrac{h}{\sqrt{3}}$. 1

(ii) Show that the volume of the pile, V m^3, is given by 1

$$V = \frac{\pi h^3}{9}.$$

(iii) Hence find the exact rate at which the height of the pile is increasing when the height of the pile is 6 metres. 2

Question 15 **(12 marks)** **Use a separate writing booklet** **Marks**

(a) Use the substitution $t = \tan\frac{x}{2}$ to show that 2

$$\frac{\cos x}{1 - \sin x} = \tan\left(45° + \frac{x}{2}\right).$$

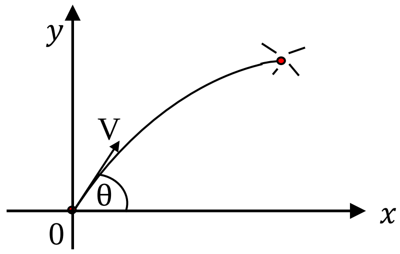
(b) A firework is launched from a point (0, 0) with a velocity of V m/s at an angle of θ to the horizontal.
This firework explodes when it reaches its maximum height.

Use the axes as shown, assume that there is no air resistance, and that the position of the firework t seconds after being launched is given by the equations:

$x = Vt\cos\theta$ and

$y = Vt\sin\theta - \frac{1}{2}gt^2$

(Do NOT prove these results)



i) Show that the maximum height reached where the firework explodes is given by $y = \dfrac{V^2\sin^2\theta}{2g}$. 2

ii) A second firework is launched from a point (0,0) with a velocity of $\frac{7V}{10}$ m/s at an angle of 2θ to the horizontal and also explodes when it reaches its maximum height.
Given that the two fireworks reach the same maximum height:

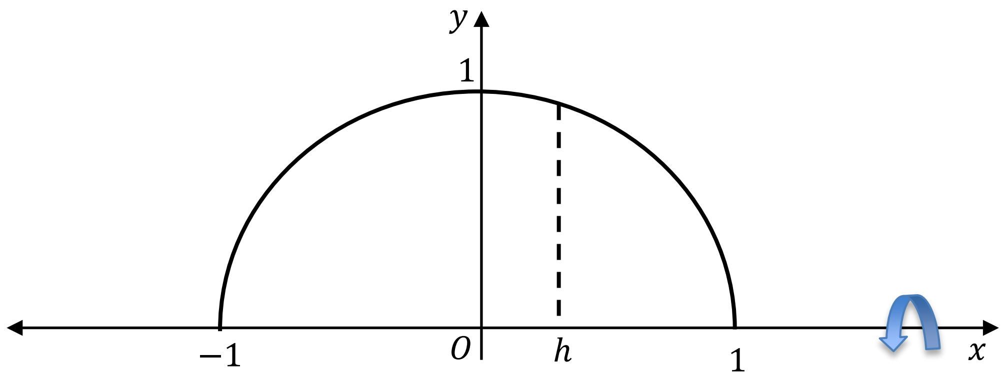
α) Show that $\cos\theta = \frac{5}{7}$. 3

β) If $v^2 = 98g$, find the maximum height reached by the two fireworks. 1

Question 15 continues on next page

Question 15 continued **Marks**

(c) The region enclosed by the semicircle $y = \sqrt{1 - x^2}$ and the x-axis is to be divided into two pieces by the line $x = h$, where $0 \leq h < 1$.



The two pieces are rotated about the x-axis to form solids of revolution. The value of h is chosen so that the volume of the solids are in the ratio 3:2.

i) Given that the volumes are in the ratio 3:2, show that **3**

$$5h^3 - 15h + 2 = 0.$$

ii) Given $h_1 = 0$ as the first approximation for h, use one application of Newton's Method to find a second approximation for h. **1**

End of examination

2018 Mathematics Extension 1 Trial HSC

MATHEMATICS EXTENSION I – MULTIPLE CHOICE

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(1) $y = 3x \Rightarrow m_1 = 3 \qquad y = -2x + 1 \Rightarrow m_2 = -2$

$\tan\theta = \left|\dfrac{3 - -2}{1 + 3(-2)}\right|$

$= \left|\dfrac{5}{-5}\right|$

$= 1$

$\therefore \theta = 45^\circ$ (D)

(2)

$\dfrac{\cos 3\alpha}{\sin\alpha} + \dfrac{\sin 3\alpha}{\cos\alpha} = \dfrac{\cos 3\alpha\cos\alpha + \sin 3\alpha \sin\alpha}{\sin\alpha\cos\alpha}$

$= \dfrac{\cos(3\alpha - \alpha)}{\frac{1}{2}\sin 2\alpha}$

$= \dfrac{2\cos 2\alpha}{\sin 2\alpha}$

$= 2\cot 2\alpha$ (D)

(3)

$\sin x - \sqrt{3}\cos x$ can be expressed in the form $R\sin(x+\alpha)$, where $R = \sqrt{1^2 + (-\sqrt{3})^2}$
$= 2$

$\therefore y = 2\sin(x+\alpha) + 1$

$\therefore$ maximum value is 3 (C)

(4) $A = 50L - L^2$

$\dfrac{dA}{dL} = 50 - 2L$ when $L = 15$, $\dfrac{dA}{dL} = 20$

Also $\dfrac{dL}{dt} = 0.08$

$\dfrac{dA}{dt} = \dfrac{dA}{dL} \times \dfrac{dL}{dt}$

$= 20 \times 0.08$ (C)

$= 1.6\ \text{cm}^2/\text{min}$

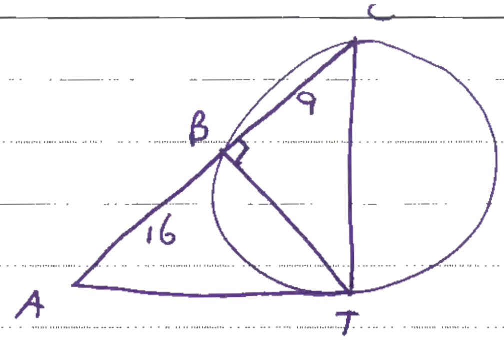
MATHEMATICS EXTENSION I – MULTIPLE CHOICE

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(5) If $(x+a)$ is a factor, $P(-a)=0$		
i.e. $-a^3+a^3-2a+1=0$		
$2a=1$		
$a=\frac{1}{2}$ (C)		
(6) $y=\pi^x$		
$=\left(e^{\ln\pi}\right)^x$		
$=e^{\ln\pi x}$		
$\frac{dy}{dx}=\ln\pi\times e^{\ln\pi x}$		
$=\ln\pi\times\pi^x$		
when $x=1$,		
$\frac{dy}{dx}=\ln\pi\times\pi$		
$=\pi\ln\pi$ (C)		
(7) $\int_0^4\frac{dx}{x^2+16}=\int_0^4\frac{dx}{x^2+4^2}$		
$=\left[\frac{1}{4}\tan^{-1}\frac{x}{4}\right]_0^4$		
$=\frac{1}{4}\left(\tan^{-1}1-\tan^{-1}0\right)$		
$=\frac{1}{4}\left(\frac{\pi}{4}-0\right)$		
$=\frac{\pi}{16}$ (B)		
(8) $-\frac{\pi}{2}\leq\sin^{-1}x\leq\frac{\pi}{2}$		
$-\pi\leq2\sin^{-1}x\leq\pi$		
$-\frac{\pi}{2}\leq2\sin^{-1}x+\frac{\pi}{2}\leq\frac{3\pi}{2}$ (D)		

MATHEMATICS EXTENSION I – MULTIPLE CHOICE

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

9

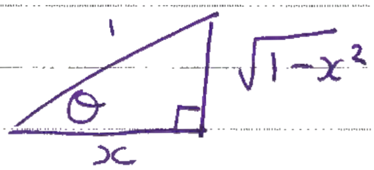


$AT^2 = 16 \times 25$ (square of tangent equals product of intercepts on secant)
$= 400$
$AT = 20$

~~CT~~ CT is a diameter ($\angle TBC = 90°$)
$\therefore CT \perp AT$ (radius perpendicular to tangent)
$\therefore AC^2 = TC^2 + AT^2$ (Pythagoras' theorem)
$TC^2 = 25^2 - 20^2$
$= 225$
$TC = 15$

B

10 let $\theta = \cos^{-1} x$



ie $\cos\theta = x$
$\therefore \sin\theta = \sqrt{1-x^2}$
$\therefore \tan\theta = \dfrac{\sqrt{1-x^2}}{x}$
$\therefore \tan(\cos^{-1}x) = \dfrac{\sqrt{1-x^2}}{x}$

B

MATHEMATICS EXTENSION I – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\lvert x-7 \rvert < \lvert 3-x \rvert$		
i.e. distance from 7 $<$ distance from 3		
		Finding $x=5$ as the critical point earned 1 mark.
$\therefore x > 5$	2	
b) let $u = x$ $\quad v = \sin^{-1}x$		
$u' = 1$ $\quad v' = \dfrac{1}{\sqrt{1-x^2}}$	1	
$\dfrac{d}{dx}(x\sin^{-1}x) = \sin^{-1}x + \dfrac{x}{\sqrt{1-x^2}}$	1	
c) $R = \left(\dfrac{2(-10)-1(-5)}{2-1}, \dfrac{2(8)-1(-1)}{2-1}\right)$	1	Correct substitution into correct formula
$= (-15, 17)$	1	
d) $\int \sec^2 x \tan x \, dx = \frac{1}{2}\tan^2 x + C$	1	
e) $\lim\limits_{x\to 0} \dfrac{\sin(\pi + x)}{x} = \lim\limits_{x\to 0} \dfrac{-\sin x}{x}$ $\left(\text{since } \sin(\pi+x) = -\sin x\right)$	1	OR $\sin\pi \dfrac{\cos x}{x} + \cos\pi \dfrac{\sin x}{x}$
$= -1$	1	
f)		
$\angle EOD = 2 \times \angle EAO$ (angle at centre twice angle at circumference) $= 50°$	1	NB: you cannot assume B lies on EO produced.
$\angle CDO = \angle EOD$ (alternate angles on parallel lines) $= 50°$	1	
$\angle ABC + \angle CDO = 180°$ (opposite angles of cyclic quadrilateral are supplementary)		
$\angle ABC = 180° - 50°$		
$= 130°$	1	

MATHEMATICS EXTENSION 1 TRIAL HSC 2018 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(a) $\boxed{x \neq -2}$

$$(x+2)^{\cancel{2}} \times \frac{4x-1}{\cancel{x+2}} \geq 1 \times (x+2)^2$$

$$(x+2)(4x-1) \geq (x+2)^2$$

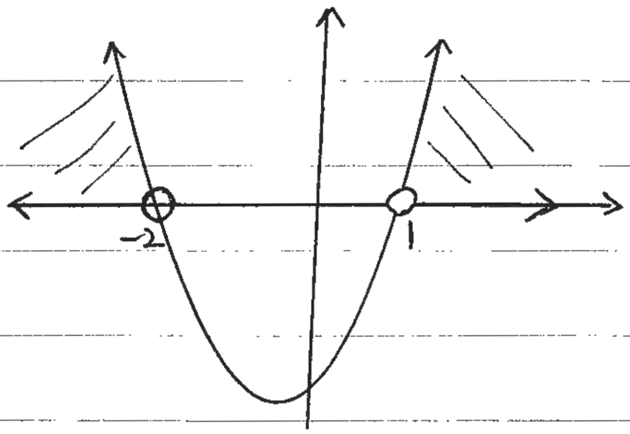
$$(x+2)(4x-1) - (x+2)^2 \geq 0$$

$$x+2\left[4x-1-(x+2)\right] \geq 0$$

$$x+2\left[4x-1-x-2\right] \geq 0$$

$$(x+2)(3x-3) \geq 0$$

$$3(x+2)(x-1) \geq 0$$



$$\therefore \underbrace{x < -2}_{1} \quad \text{OR} \quad \underbrace{x \geq 1}_{1}$$

- this question was mostly well done.
- be careful with algebra!
- make the statement about the denominator $x \neq -2$!!! Hence $x < -2$ is the solution not $x \leq -2$.

1 for progress to answer

- be careful how you draw your parabola ie when its $\cup$ OR $\cap$ depending on your working.

3 provides correct solution with working

2 correctly provides $x \leq -2$ OR $x \geq 1$ with working

1 demonstrates some progress

MATHEMATICS EXTENSION 1 TRIAL HSC 2018 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(b) $u = x+1 \Rightarrow x = u-1$		
$\frac{du}{dx} = 1$ when $x=0$, $u=1$		
$\therefore du = dx$ $x=3$, $u=4$		
(1) for these steps		
$\int_0^3 \frac{x}{\sqrt{x+1}}\,dx$		
$= \int_1^4 \frac{u-1}{\sqrt{u}} \cdot du$		
$= \int_1^4 \frac{u}{u^{\frac{1}{2}}} - \frac{1}{u^{\frac{1}{2}}}\,du$		(1) for these lines or equivalent
$= \int_1^4 u^{\frac{1}{2}} - u^{-\frac{1}{2}}\,du$		
$= \left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^4$ — (1) for integrating correctly		
$= \frac{2}{3}(4)^{\frac{3}{2}} - 2(4)^{\frac{1}{2}} - \left(\frac{2}{3} - 2\right)$		
$= \frac{16}{3} - 4 - \left(\frac{2}{3} - 2\right)$		
$= \frac{8}{3}$ — (1) for correct answer		
• this question was well done!		

MATHEMATICS EXTENSION 1 TRIAL HSC 2018 – QUESTION 12		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(c) $v^2 = 6 + 4x - 2x^2$

(i) two extremes occur when $v = 0$:

$$-2x^2 + 4x + 6 = 0$$

$$-2(x^2 - 2x - 3) = 0$$

$$(x-3)(x+1) = 0$$

$$\therefore x = 3 \text{ or } x = -1$$

$\therefore$ particle oscillates between -1 and 3. — ① provides both correct answers

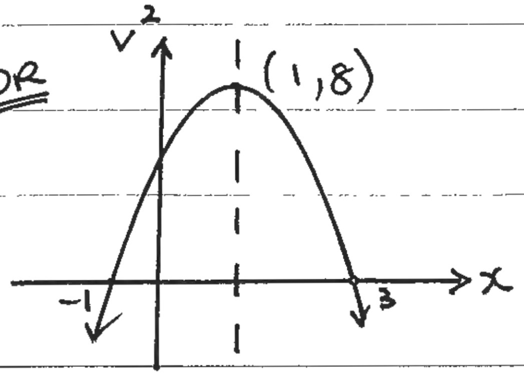
(ii) maximum speed occurs at the centre of motion ie when $x = 1$

$$v^2 = 6 + 4(1) - 2(1)^2$$
$$= 6 + 4 - 2$$
$$= 8$$

max. speed $= |v| = \sqrt{8}$

$= 2\sqrt{2}$ m/s — ① provides correct answer

OR



$v^2 = 8$

$\therefore$ max. speed $= |v|$

$= \sqrt{8}$

$= 2\sqrt{2}$ m/s

MATHEMATICS EXTENSION 1 TRIAL HSC 2018 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(iii) $a = \ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$		
$= \frac{d}{dx}(3+2x-x^2)$		
$\therefore a = 2-2x$ —	1 provides correct answer	
• this question was well done		
• watch calculations		
• no half marks!		
(d) $2\sin^2 x - 1 = 0$		• mostly well done!
$2\sin^2 x = 1$		• use reference sheet
$\sin^2 x = \frac{1}{2}$		• don't forget where $n \in \mathbb{Z}$ or where n is an integer.
$\sin x = \pm\frac{1}{\sqrt{2}}$		• no $\frac{1}{2}$ marks.
For $\sin x = \frac{1}{\sqrt{2}}$:		
$x = n\pi + (-1)^n \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$	1 mark	
$x = n\pi + (-1)^n \frac{\pi}{4}$ where n is an integer.		
For $\sin x = -\frac{1}{\sqrt{2}}$:		
$x = n\pi + (-1)^n \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$	1 mark	
$x = n\pi + (-1)^n \left(-\frac{\pi}{4}\right)$ where n is an integer.		
OR $x = n\pi - (-1)^n \frac{\pi}{4}$		

MATHEMATICS EXTENSION 1 TRIAL HSC 2018 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR (d) $2\sin^2 x - 1 = 0$		
$2\sin^2 x = 1$		
$\sin^2 x = \frac{1}{2}$	1	-225° 135° | -315° 45°
$\sin x = \pm\frac{1}{\sqrt{2}}$		225° -135° | 315° -45°
(ie all 4 quadrants)		
$\therefore x = n\pi \pm \frac{\pi}{4}$ where $n \in \mathbb{Z}$ $\rightarrow$ 1		
OR $2\sin^2 x - 1 = 0$		• $\cos 2x = 1 - 2\sin^2 x$
$-\cos 2x = 0$		
$\cos 2x = 0$		
$2x = 2n\pi \pm \cos^{-1} 0$		
$2x = 2n\pi \pm \frac{\pi}{2}$ $\rightarrow$	1	mark
$x = \frac{1}{2}\left(2n\pi \pm \frac{\pi}{2}\right)$		
$\therefore x = n\pi \pm \frac{\pi}{4}$ where $n \in \mathbb{Z}$		
$\rightarrow$	1	mark
NOTE: $\dfrac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$		
& $\frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$		

MATHEMATICS EXTENSION 1 TRIAL HSC 2018 – QUESTION 12		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

OR The really long way !!!

$$2\sin^2 x - 1 = 0$$

$$2\sin^2 x = 1$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm\frac{1}{\sqrt{2}}$$

For $\sin x = +\frac{1}{\sqrt{2}}$:

$x = \left(\sin^{-1}\frac{1}{\sqrt{2}}\right) + 2n\pi$ OR $x = \left(\pi - \sin^{-1}\frac{1}{\sqrt{2}}\right) + 2n\pi$

$x = \frac{\pi}{4} + 2n\pi$ $\quad$ $x = \left(\pi - \frac{\pi}{4}\right) + 2n\pi$

$x = \frac{3\pi}{4} + 2n\pi$

1 mark

For $\sin x = -\frac{1}{\sqrt{2}}$:

$x = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) + 2n\pi$ OR $x = \pi - \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) + 2n\pi$

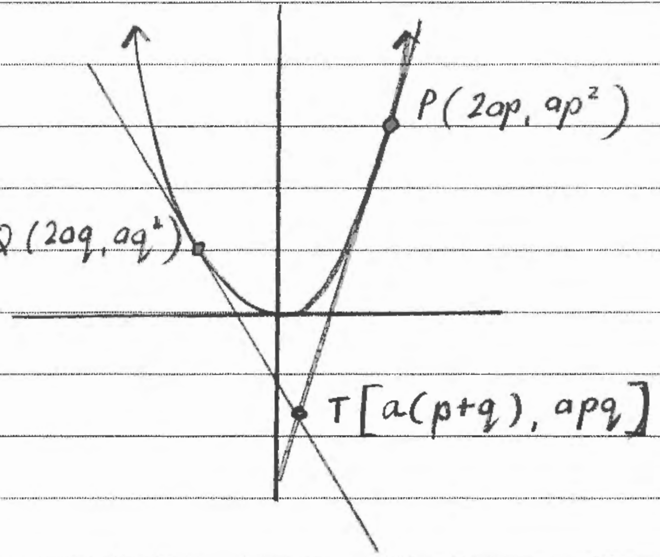
$x = -\frac{\pi}{4} + 2n\pi$ $\quad$ $= \pi - -\frac{\pi}{4} + 2n\pi$

$x = \frac{5\pi}{4} + 2n\pi$

1 mark

where n is an integer !

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a)		
		
$y = \frac{x^2}{4a}$ $y' = \frac{x}{2a}$ $x = 2ap$ $m = \frac{2ap}{2a}$ $m = p$	1	
$y - ap^2 = p(x - 2ap)$ $y - ap^2 = px - 2ap^2$ $y = px - ap^2 \quad \ldots 1$	1	
(ii) Similarly $y = qx - aq^2 \quad \ldots 2$		
Solve 1 and 2 simultaneously $px - ap^2 = qx - aq^2$ $px - qx = ap^2 - aq^2$ $(p-q)x = a(p^2 - q^2)$ $x = \frac{a\cancel{(p-q)}(p+q)}{\cancel{(p-q)}}$ $x = a(p+q)$	1	

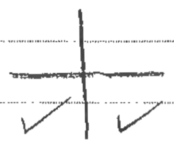
MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$y = px - ap^2$		
$y = p \times a(p+q) - ap^2$		
$= ap^2 + apq - ap^2$		
$y = apq$	①	
$\therefore T\left[a(p+q),\ apq\right]$		
(iii) $\tan\theta = \left\lvert \frac{m_2 - m_1}{1 + m_1 m_2} \right\rvert$ $\quad m_1 = p \quad m_2 = q$		
$\tan 45° = \left\lvert \frac{p-q}{1+pq} \right\rvert$		They needed to have $\overline{\tan 45°}$ not just "1"
$\left\lvert \frac{p-q}{1+pq} \right\rvert = 1$ Now $p > q > 0$ $\therefore p - q > 0$ and $1 + pq > 0$		Please stress that removing the absolute value signs should be explained.
$\frac{p-q}{1+pq} = 1$	①	
$p - q = 1 + pq$		

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(iv) $x^2 - 4ay$ $T[a(p+q), apq]$		
NOTE The question actually guided you :		
$x = a(p+q)$ $\quad$ 4a × $\quad y = apq$ ×4a		
$x^2 = a^2(p+q)^2 \ldots (1)$ $\quad 4ay = 4a^2pq \ldots (2)$		
(1) − (2)		
$x^2 - 4ay = a^2(p+q)^2 - 4a^2pq$		
This is also the substitution the question asked for		
SOLUTION		
$x^2 - 4ay = [a(p+q)]^2 - 4a(apq)$		You must do the substitution to gain full marks. (1)/2 if correct solution without substitution.
$= a^2(p+q)^2 - 4a^2pq$		
$= a^2(p^2+2pq+q^2) - 4a^2pq$		
$= a^2(p^2+2pq+q^2-4pq)$		
$= a^2(p^2-2pq+q^2)$		
$= a^2(p-q)^2$	1	Note - only ½ if they had =0 and zer if eliminated a^2.
From (iii) $p-q = 1+pq$		
$= a^2(1+pq)^2$ $\quad$ Now $y = apq$	½	using identity (iii)
$pq = \frac{y}{a}$	½	subbing in $pq = \frac{y}{a}$
$= a^2\left(1+\frac{y}{a}\right)^2$		
$x^2 - 4ay = a^2\left(1 + \frac{2y}{a} + \frac{y^2}{a^2}\right)$		

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$x^2 - 4ay = a^2 + 2ay + y^2$		
$x^2 - y^2 = a^2 + 6ay$	$\frac{1}{2}$	correct answer
b) $2\sin x - 3\cos x = R\cos(x+\alpha)$		
$2\sin x - 3\cos x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
$2 = -R\sin\alpha$ $-3 = R\cos\alpha$		Many missed negative on $R\sin\alpha = -2$
$R\sin\alpha = -2$ ($\frac{1}{2}$) $R\cos\alpha = -3$ ($\frac{1}{2}$)		
 		
Overlap quad 3		Many forgot this!
$R^2\sin^2\alpha + R^2\cos^2\alpha = 4 + 9$		
$R^2(\sin^2\alpha + \cos^2\alpha) = 13$		
$R^2 = 13$ $R > 0$		
$R = \sqrt{13}$	$\frac{1}{2}$	
$\frac{R\sin\alpha}{R\cos\alpha} = \frac{-2}{-3}$		
$\tan\alpha = \frac{2}{3}$ * QUAD 3 *		
$\alpha = 0.588\ldots$		RADIANS!
$\alpha = \pi + 0.588$		(no penalty
$\alpha = 3.73$	1	but PLEASE remember)
$\sqrt{13}\cos(x + 3.7)$	$\frac{1}{2}$	- Many forgot to write this

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) $100 = 80 + Ae^{0.1t}$		
$t = 0$		
$100 = 80 + Ae^{0}$		
$A = 20$ —	$\frac{1}{2}$	Evaluating A
$N = 80 + 20e^{0.1t}$		
$200 = 80 + 20e^{0.1t}$		
$120 = 20e^{0.1t}$		
$6 = e^{0.1t}$		
$t = \frac{\ln 6}{0.1}$		(some halves for very small numerical error)
$t = 17.9175\ldots$ —	1	correct value of t
$\underbrace{t=0 \quad\quad t=1}_{\text{first yr}} \quad \underbrace{t=1 \quad\quad t=2}_{\text{2nd yr}} \quad t=3 \quad \cdots \quad \underbrace{t=17 \quad t=18}_{18^{th}\text{ yr}}$ 2000, 2001, 2002, 2003, ….. 2017, 2018		
During 2017 or in the 18^{th} year. —	$\frac{1}{2}$	Correct year.

MATHEMATICS EXTENSION I – QUESTION 14 TRIAL 2018

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
pg1.		
(i) Use the sum of the terms of an Arithmetic sequence to show that $(1+2+3+\dots+n)^2 = \frac{1}{4}n^2(n+1)^2$	1	Generally well done
$[1+2+3+\dots+n]$ is an A.S with $a=1$ $d=1$ $n=n$ $L=n$		(no $\frac{1}{2}$ marks)
$S_n = \frac{n}{2}[a+L]$ $= \frac{n}{2}[1+n]$ $= \frac{n}{2}[n+1]$ —— ①		
OR $S_n = \frac{n}{2}[2a+(n-1)d]$ $= \frac{n}{2}[2\times 1+(n-1)\times 1]$ $= \frac{n}{2}[2+n-1]$ $= \frac{n}{2}[1+n]$ $= \frac{n}{2}[n+1]$ —— ✓		
substituting ① in		
$(1+2+3+\dots+n)^2 = \left[\frac{n}{2}(n+1)\right]^2$ $= \frac{n^2}{4}(n+1)^2$ $= \frac{1}{4}(n^2)(n+1)^2$		
$\therefore (1+2+3+\dots+n)^2 = \frac{1}{4}(n^2)(n+1)^2$ as required		
(ii) Prove by induction $1^3+2^3+\dots+n^3 = (1+2+\dots+n)^2$ for all integers $n\geq 1$		
Step 1 Show the result is true when $n=1$.	$\frac{1}{2}$	
when $n=1$ LHS $= 1^3$ $= 1$ RHS $= 1^2$ $= 1$		
LHS = RHS = 1		
$\therefore$ the result is true when $n=1$.		
Step 2 Assume the result is true for some integer k (where $k\geq 1$)	$\frac{1}{2}$	some students used part (i) at this stage justifying and replacing
ie Assume $1^3+2^3+3^3+\dots k^3 = (1+2+3+\dots+k)^2$		$(1+2+3+\dots k)^2$ by $\frac{1}{4}k^2(k+1)^2$

MATHEMATICS EXTENSION I – QUESTION 14 TRIAL 2018

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

pg 2

Step 3) Prove the result is true for $n = k+1$ assuming it is true for $n = k$.

ie Prove

$$1^3 + 2^3 + 3^3 + \ldots (k+1)^3 = [1 + 2 + 3 + \ldots (k+1)]^2$$

$$LHS = 1^3 + 2^3 + 3^3 + \ldots (k+1)^3$$

$$= \underbrace{1^3 + 2^3 + 3^3 + \ldots (k)^3}_{S_k} + \underbrace{(k+1)^3}_{T_{k+1}}$$

$$= \frac{1}{4} k^2 (k+1)^2 + (k+1)^3$$

(using part (i) $1 + 2 + 3 + \ldots + k^2 = \frac{1}{4}(k^2)(k+1)^2$)

$$= \frac{(k+1)^2}{4} \left[k^2 + 4(k+1) \right]$$

$$= \frac{(k+1)^2}{4} \left[k^2 + 4k + 4 \right]$$

$$= \frac{1}{4}(k+1)^2 (k+2)^2$$

$$= [1 + 2 + 3 + \ldots (k+1)]^2$$

since $\frac{1}{4}(k^2)(k+1)^2 = 1 + 2 + \cdots k^2$

$= RHS$

$\therefore$ the result is true when $n = k$ if it is true when $n = k+1$.

Step 4: If the statement is true for $n = k$, then it is true for $n = k+1$ since the result is true for $n = 1+1 = 2$ and for $n = 2+1 = 3$. and so on for all positive integers n.

Marker's comments: some students used part (i) earlier in this stage also.

MATHEMATICS EXTENSION I – QUESTION 14 TRIAL 2018

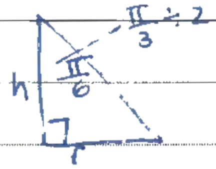
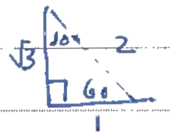
pg3

(b)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(i) $\begin{array}{r} 2x^2 - 4x - 2 \\ x^2-3x+1 \overline{) 2x^4 - 10x^3 + 12x^2 + 2x - 3} \\ \underline{2x^4 - 6x^3 + 2x^2} \\ -4x^3 + 10x^2 + 2x - 3 \\ \underline{-4x^3 + 12x^2 - 4x} \\ -2x^2 + 6x - 3 \\ \underline{-2x^2 + 6x - 2} \\ -1 \end{array}$	2	−1 per error if had 2 errors but looked basically correct gave ½ instead of 0
(ii) $2x^4 - 10x^3 + 12x^2 + 2x - 3$ $= (x^2 - 3x + 1)(2x^2 - 4x - 2) - 1$ OR $= (x^2 - 3x + 1)([2(x^2 - 2x - 1)]) - 1$	1	
(iii) $f(x) = 2x^4 - 10x^3 + 12x^2 + 2x - 3$ $g(x) = x^2 - 3x + 1.$ from (ii). $f(x) = g(x)\,q(x) - 1$ let k be a zero of $g(x)$ then $g(k) = k^2 - 3k + 1.$ and $k^2 - 2k + 1 = 0.$ —— ① $\therefore g(k) = 0.$ $f(k) = g(k)\,q(k) - 1$ = —— ② {sub ① in ② $= 0\,q(k) - 1$ $= 0 - 1$ $= -1$ $\therefore$ k is <u>not a zero</u> of $f(x)$. since $f(k) = -1.$ $\therefore$ $f(x)$ and $g(x)$ have no zeros in common		

pg 4

MATHEMATICS EXTENSION I – QUESTION 14 TRIAL 2018

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(c) (i)  	1	most students that had difficulty didn't get the angle, $\frac{\pi}{6}$, correct.
$\tan 30° = \frac{1}{\sqrt{3}}$.		
$\tan\frac{\pi}{6} = \frac{r}{h}$		
$\frac{1}{\sqrt{3}} = \frac{r}{h}$		
$\therefore r = h \times \frac{1}{\sqrt{3}}$		
$r = \frac{h}{\sqrt{3}} \longleftarrow ①$		
(ii) Show $V = \frac{\pi h^3}{9}$	1.	well done
$V_{cone} = \frac{1}{3}\pi r^2 h \longleftarrow ②$		
sub ① in ② $= \frac{1}{3} \times \pi \times \left(\frac{h}{\sqrt{3}}\right)^2 h$		
$= \frac{1}{3} \times \pi \times \frac{h^2}{3} \times h$		
$\therefore V = \frac{\pi h^3}{9}$ as required.		
(iii) $\frac{dV}{dt} = 1.5\ m^3/s$. $\quad \frac{dh}{dt} = ?$		many methods. 1 or 2 marks.
$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$		need to show basic understanding and $\frac{dV}{dh}$ to get any marks.
$V = \frac{\pi h^3}{9}$		
$\frac{dV}{dh} = \frac{3\pi h^2}{9}$		
$\frac{dV}{dh} = \frac{\pi h^2}{3}$.		

pg5

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$

$$1.5 = \frac{\pi h^2}{3} \times \frac{dh}{dt}$$

when $h = 6$

$$1.5 = \frac{\pi \times 6^2}{3} \times \frac{dh}{dt}$$

$$1.5 \times 3 = 36\pi \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4.5}{36\pi}$$

$$= \frac{1}{8\pi} \quad m^3/s.$$

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) LHS $= \dfrac{\cos x}{1-\sin x}$		
$= \dfrac{\frac{1-t^2}{1+t^2}}{1-\frac{2t}{1+t^2}}$		
$= \dfrac{\frac{1-t^2}{1+t^2}}{\frac{1+t^2-2t}{1+t^2}}$ $\left[= \dfrac{1-t^2}{1+t^2-2t}\right]$	½	
$= \dfrac{1-t^2}{(t-1)^2}$		
$= \dfrac{(1-t)(1+t)}{(t-1)^2}$	½	
$= \dfrac{-(t-1)(1+t)}{(t-1)^{\cancel{2}}}$		
$= \dfrac{-(1+t)}{t-1}$		
$= \dfrac{-(1+t)}{-(1-t)}$	½	
$= \dfrac{1+t}{1-t}$		
RHS $= \tan\left(45 + \dfrac{x}{2}\right)$		
$= \dfrac{\tan 45 + \tan\frac{x}{2}}{1 - \tan 45 \tan\frac{x}{2}}$	½	
$= \dfrac{1+t}{1-t} =$ LHS		2

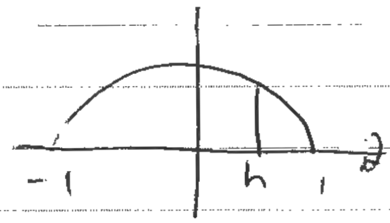
MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) i) Max height occurs when vertical velocity $\dot{y} = 0$.		
$\dot{y} = v\sin\theta - gt$		
$\therefore v\sin\theta - gt = 0$		
$t = \frac{v\sin\theta}{g}$ sub in y for Max	1	
Sub in y for Max height		
$y = v\left(\frac{v\sin\theta}{g}\right)\sin\theta - \frac{g}{2}\left(\frac{v\sin\theta}{g}\right)^2$	1	
$= \frac{v^2\sin^2\theta}{g} - \frac{v^2\sin^2\theta}{2g}$		
$= \frac{2v^2\sin^2\theta - v^2\sin^2\theta}{2g}$		2
$= \frac{v^2\sin^2\theta}{2g}$ --- (1)		
ii) For the second firework the max height reached given $v = \frac{7v}{10}$ and at an angle of 2θ is:		Some students tried to derive this expression but made a few error along the way. Care is needed when simplifying.
$y = \frac{\left(\frac{7v}{10}\right)^2\sin^2 2\theta}{2g}$ --- (2)	½	
But as both fireworks have the same maximum height: (1) = (2)		
$\frac{v^2\sin^2\theta}{2g} = \frac{\left(\frac{7v}{10}\right)^2\sin^2 2\theta}{2g}$	←	1 mark to equate the 2 max heights
$\cancel{v^2}\sin^2\theta = \frac{49v^2}{100}\sin^2 2\theta$		

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
ii) α) Cont'd		
$100\sin^2\theta = 49\sin^2 2\theta$		
$100\sin^2\theta = 49(\sin 2\theta)^2$		
$= 49(2\sin\theta\cos\theta)^2$	1	
$= 49 \times 4 \times \sin^2\theta\cos^2\theta$		
$\frac{100}{196} = \cos^2\theta$	½	
ie, $\cos^2\theta = \frac{25}{49}$		3
$\cos\theta = \frac{5}{7}$ as θ is acute		
ii) β) The max height is:		
$y = \frac{V^2\sin^2\theta}{2g}$		
as $V^2 = 98g$ then		Many student squared 98g and evidently did not receive the correct value for y.
$y = \frac{(98g) \times \sin^2\theta}{2g}$	½	
$= 49 \times \sin^2\theta$		
$= 49 \times (1 - \cos^2\theta)$		
$= 49 \times \left(1 - \frac{25}{49}\right)$	½	
$= 24$		
∴ the maximum height reached by the two fireworks is 24m.		1

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i) 		Remember: When you are finding the Volume you need to use the formula correctly ie $V = \pi \int y^2 dx$ Many students did not use the π in the formula and lost ½ mark
$V_1 = \pi \int_{-1}^{h} \left(\sqrt{1-x^2}\right)^2 dx$		
$= \pi \int_{-1}^{h} 1 - x^2 \, dx$		
$= \pi \left[x - \frac{x^3}{3} \right]_{-1}^{h}$		
$= \pi \left[h - \frac{h^3}{3} - \left(-1 - \frac{(-1)^3}{3} \right) \right]$		
$= \pi \left[h - \frac{h^3}{3} + 1 - \frac{1}{3} \right]$		
$= \pi \left(h - \frac{h^3}{3} + \frac{2}{3} \right)$		
$V_2 = \pi \int_{h}^{1} 1 - x^2 \, dx$		1½ marks were given for 2 correct volumes V_1 & V_2.
$= \pi \left[x - \frac{x^3}{3} \right]_{h}^{1}$		
$= \pi \left[1 - \frac{1}{3} - \left(h - \frac{h^3}{3} \right) \right]$		1 mark for using the ratio $\frac{V_1}{V_2} = \frac{3}{2}$ correctly.
$= \pi \left(\frac{2}{3} - h + \frac{h^3}{3} \right)$		
But $\frac{V_1}{V_2} = \frac{3}{2}$		
So $2V_1 = 3V_2$		

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i) Cont'd		
$2\left(h - \frac{h}{3} + \frac{2}{3}\right) = 3\left(\frac{2}{3} - h + \frac{h^3}{3}\right)$		
$2h - \frac{2h^3}{3} + \frac{4}{3} = 2 - 3h + h^3$		½ mark for correct simplification to establish answer.
$6h - 2h^3 + 4 = 6 - 9h + 3h^3$		
$5h^3 - 15h + 2 = 0$		3
Method 2		
$V_1 = \pi \int_h^1 1 - x^2 \, dx$		
$\vdots$		
$= \pi\left(\frac{2}{3} - h + \frac{h^3}{3}\right)$		
This volume is $\frac{2}{5}$ of the volume of the sphere.		
i.e. $V = \frac{2}{5} \times \frac{4}{3}\pi r^3$		
$= \frac{8}{15}\pi$		
So $V = V_1$		
$\frac{8}{15}\pi = \pi\left(\frac{2}{3} - h + \frac{h^3}{3}\right)$		
$8\pi = 10 - 15h + 5h^3$		
$5h^3 - 15h + 2 = 0$		

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c ii) Let $f(x) = 5h^3 - 15h + 2$		
$f'(x) = 15h^2 - 15$		
Taking $h_1 = 0$		
$h_2 = h_1 - \frac{f(h_1)}{f'(h_1)}$		
$= 0 - \frac{f(0)}{f'(0)}$		
$= 0 - \frac{2}{-15}$	½	
$= \frac{2}{15}$	½	1